

利用棣美弗定理導出正、餘弦函數的三倍角公式
(資料來源:龍騰教師手冊)

$$\begin{aligned}\text{因為 } (\cos \theta + i \sin \theta)^3 &= \cos^3 \theta + 3 \cos^2 \theta \cdot i \sin \theta + 3 \cos \theta \cdot (i \sin \theta)^2 + (i \sin \theta)^3 \\ &= (\cos^3 \theta - 3 \cos \theta \sin^2 \theta) + i(3 \cos^2 \theta \sin \theta - \sin^3 \theta),\end{aligned}$$

又由棣美弗定理, 知 $(\cos \theta + i \sin \theta)^3 = \cos 3\theta + i \sin 3\theta$,

所以

$$\begin{aligned}\cos 3\theta &= \cos^3 \theta - 3 \cos \theta \sin^2 \theta = \cos^3 \theta - 3 \cos \theta (1 - \cos^2 \theta) \\ &= 4 \cos^3 \theta - 3 \cos \theta .\end{aligned}$$

$$\begin{aligned}\sin 3\theta &= 3 \cos^2 \theta \sin \theta - \sin^3 \theta = 3(1 - \sin^2 \theta) \sin \theta - \sin^3 \theta \\ &= 3 \sin \theta - 4 \sin^3 \theta .\end{aligned}$$